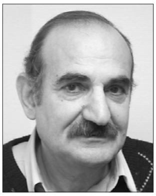
VATTEN 61: 193–200. Lund 2005

SOME NOTES ON HYDRAULICS AND A MATHEMATICAL DESCRIPTION OF SLOW SAND FILTRATION

by HUSAM SALEH JABUR,[1] JONAS MÅRTENSSON[1] and GEZA ÖLLÖS[2]
1 SWECO Viak AB, Jönköping Sweden
e-mail: jonas.martensson@sweco.se
2 Technical University of Budapest, Hungary



Abstract

Reliable mathematical models that can describe the mechanisms in Slow Sand Filters, SSF, represented by different filter parameters are yet absent. The extremely complicated interactions between physical, chemical, biological and hydraulic conditions are shown in a simplified model.

An attempt to apply the rapid filtration empirical formulas for SSF is described in detail. The need for further research and empirical surveys is obvious to determine fundamental filter parameters.

The negative effect on the head loss caused by hydraulic negative pressure, "vacuum", is presented.

Key words – Hydraulics, modelling, theory, slow sand filtration, head loss, negative pressure.

Introduction

Water trickles through the soil interstice during the natural hydrological cycle. The management of water resources, as a planned activity of man, often result in local or regional changes in this natural hydraulic cycle.

When surface waters, that usually contains impurities, trickles through a fine-grained aquifer, filtration will occur. As a consequence, interaction forces and different effects are formed between the ground water and the huge surface area of the particles in the aquifer. The water quality will then change to a smaller or larger extent depending on the actual parameters studied. This process to improve the water quality is used in the artificial ground water recharge, bank infiltration etc.

Consequently it is fair to theoretically consider the subterranean seepage as a water treatment process. This could be of importance, since the water quality changes in both positive and negative direction depending on the composition of the aquifer.

Hydraulic Principles of filtration

When trying to describe the subterranean seepage by kinematical means, the trickling in most cases can be considered as a quasi-permanent state. The liquid is incompressible, as well as the solids in the aquifer. The seepage space is homogeneous and isotropic. The water will completely fill the tension free porous system, and thus the aquifer consists of two phases.

The two available hydrodynamic fundamental equations to describe the status are the equation of continuity and the Navier-Stokes equation (Huisman and Olsthoorn 1983, Kovacs 1972).

The equation of continuity for an incompressible system is:

$$\partial v_x/\partial x + \partial v_y/\partial y + \partial v_z/\partial z = \text{div } v = 0 \qquad (1)$$

where v is the flow velocity (m/s). The equation describes the flow gradients in a Cartesian coordinate system.

Since the only resistance force is the viscous force, the conservation of energy expressed by Navier-Stokes equation for laminar flow is:

$$1/\varphi g \cdot \partial v/\partial t = -\text{grad } (z + p/\gamma) - v/k \qquad (2)$$

where φ is the porosity (%), g the acceleration of gravity (m/s^2), $(z + p/\gamma)$ the piezometric head (m) and k the coefficient of permeability (m/s).

At steady motion the equation can be expressed as:

$$v = -k \text{ grad } (z + p/\gamma) = -k \text{ grad } h_w \quad (3)$$

in which h_w is the piezometric head (m).

In practice though, the Darcy's law is usually applied in most water-supply exercises:

$$v = k \cdot I \quad (4)$$

where v is flow velocity (m/s), k the coefficient of permeability (m/s) and I the hydraulic gradient.

The dynamical conditions to apply Darcy's law are that the accelerating force is the gravity and the resistance force is the viscous force.

The upper limit for the application of Darcy's law is characterized by a Reynolds number, which expresses the ratio between the inertia and the viscous forces. It should be lower than a certain numerical value, approx. Re = 2–5:

$$Re = d \cdot v/\nu \quad (5)$$

in which d is the grain effective diameter (m), v flow velocity (m/s) and ν Water viscosity (m^2/s).

The lower limit is expressed by the ratio between the critical hydraulic gradient I_C and the limitation gradient I_0 as the following (Kovacs 1972):

$$I_C > 12\, I_0 \quad (6)$$

In this case the adhering force as a resistance force is negligible compared to the viscous force.

In an application of Darcy's law at inhomogeneous and anisotropic conditions, the changes in values of the k parameter in space and direction must be calculated.

The above equations presume's that the seepage water is clean and no impurities enter the porous system. Thus the interaction forces between the surface of the particles and the seepage water are unchanged in time and space. The internal and external conditions of the seepage system are constant and the k parameter is constant in time.

These basic hydraulic equations are in principle valid for filtration. In slow sand filtration Reynolds number is Re < 2 and the critical hydraulic gradient $I_C > 12\, I_0$. However in this case it should be considered that the trickling water through the porous system also contain impurities as suspended materials, colloids, flocks, dissolved materials, living and dead organisms etc.

During its passage the impurities, especially the solids, are brought into contact with the surface of the sand grains and held in position there by the effect of the transport mechanisms, which principally involves straining, sedimentation and adsorption (Woodward and Ta, 1988).

As the impurities are removed from the trickling flow and deposited in the soil various typical filtration parameters, d, φ, σ, I, v, Re etc, are changed in time and space due to clogging. The hydraulic gradient I = dh/dl refer only to a certain cross-section.

Theory of Filtration

In rapid filtration the physical processes usually are dominating, while the biological processes are negligible. The mathematical relations between the filtration parameters are relatively well characterized.

Various mathematical models of rapid filtration have been developed during the past decades. These models can be divided in two (Ives, 1969). One part is related to clarification of suspensions. The other part is relating to the rise in head loss due to filter clogging.

It is obvious that no accepted mathematical model has been obtained that correlates all the physical variables and filtration parameters. The complexity of water quality, fluid motions and filtration processes make it difficult to be predictive.

Iwasaki first formulated a clarification mathematical equation in 1937, where he expressed the removal rate of the concentration of suspended solids from a flow as proportional to the local concentration. This widely accepted empirical formulae (Camp, 1964, Deb, 1969) is:

$$\partial C/\partial L = -\lambda \cdot C \quad (7)$$

where:

C is the concentration of particles (in number of particles/cm^3)

L is the distance from top of the filter bed from which C is measured (m).

λ is a filter coefficient (cm^{-1})

By the introduction of a dimensionless specific deposit coefficient σ, Iwasaki developed a second relationship as:

$$\partial C/\partial L = 1/v \cdot \partial\sigma/\partial t \quad (8)$$

where v is the flow velocity (m/s)

He suggested the relationship of λ with σ as:

$$\lambda = \lambda_0 + c\sigma \quad (9)$$

where:

λ_0 is the initial filter coefficient (cm^{-1})

c is a constant (cm^{-1})

Investigators in this field all agree that λ varies with σ. They have developed several kinds of modelling approaches to cover the whole range of filtration. (Mints 1966, Ives 1969, Matsui and Tambo 1995). Ives (1969), suggested that:

$$\lambda = \lambda_0 + a\sigma - b\sigma^2/(\varphi_0 - \sigma) \quad (10)$$

where:

a is a filtration parameter representing the positive effect of deposits on the filter efficiency and water quality early in the filtration process (cm^{-1}).

b is a filtration parameter representing the negative effect of deposits on the filter efficiency and water quality late in the filtration process (cm^{-1}).

φ_0 is the porosity of clean bed (%)

In the equation above the λ_0 term represent the clean filter bed. The term $a\sigma$ show that, in the beginning of the filtration run, the filter coefficient λ would increase linearly with the specific deposit σ. The negative term represent the decrease of the λ coefficient towards the end of filter run.

Many models also emphasize the key role of break-through curves in rapid filtration design (Adin and Rebhun, 1977).

Theory of Slow Sand Filters

By slow sand filtration, in addition to physical and chemical processes, the biological processes are essential. A part of the deposited material is converted to other forms, by assimilation into biomass, and by biological degradation to dissimilation products, such as minerals and gases. On top of the filter media photosynthesis yields a further input of particulate and organic materials. As a consequence certain changes occur on the surface of the media grains influencing e.g. forces and filter parameters. The above discussed rapid filtration relationships can thus not be applied in an analogous way in slow sand filtration.

Jabur, 1976, suggests that for slow sand filtration the dimensionless coefficient σ should be divided into two parts (Jabur 1976, Öllös 1987):

$$\sigma = \sigma_1 + \sigma_2 \quad (11)$$

where:

σ_1 is the inconvertible specific deposit

σ_2 is the convertible specific deposit

By the introduction of S, a slow sand filter parameter, the equation (10) for slow sand filters is written as follows:

$$\lambda = \lambda_0 + a(\sigma_1 + \sigma_2) - b(\sigma_1 + \sigma_2)^2/[\varphi_0 - (\sigma_1 - \sigma_2)] + S\sigma_2 \quad (12)$$

where

S is a slow sand filter time and depth dependent parameter (m^{-1})

$S\sigma_2$ is the changes of σ due to the slow sand filtration processes

In slow sand filters no break-through of the filter media normally occurs at proper hydraulic operation and with absence of negative pressure. This is caused by the strong adsorption capacity and relatively low filtration velocity. The negative term suggested in equation (12) is thus negligible. The relationships of λ with σ for the whole range of slow sand filters is then the following:

$$\lambda = \lambda_0 + a\sigma + S\sigma_2 \quad (13)$$

$$-\partial C/\partial L = (\lambda_0 + a\sigma + S\sigma_2) \cdot C \quad (14)$$

The equations 13 and 14 shows, that by adding the $S\sigma_2$ term the relationship of λ with σ in slow sand filtration is very complicated compared to rapid filtration. One complication is to measure the S parameter.

The mathematical equations have thus up today only a "philosophical" importance, because of the variety of mechanisms and the complicity to determine the effect of the different filtration parameters. The lack of late references illuminates this.

Slow sand filtration is a simple technology with respect to the filter construction, but is shown to be extremely complicated in its function with respect to physical, chemical, biological and hydraulic behaviour.

A simplified system model is shown in figure 1. In the model the relations between different treatment processes and the most important parameters are shown (Jabur, Mårtensson, 1999).

In the last decades new efforts have been made to include the relation between different micro organisms and to take into consideration the importance of the upper thin layer, Schmutzdecke, in modelling of slow sand filters (Woodward and Ta 1988, Ojha and Graham 1996).

While the mechanisms of slow sand filters are somewhat known, quantitative theories faces problems caused by the complicity, and the difficulties to measure the relevant specific variables (Woodward and Ta 1988).

Head Loss

The head-loss diagram of slow sand filters, similar to rapid filters, consists of two components. The homogeneous clean filter media resistance and the extra resistance due to clogging.

The clean media resistance at a certain depth h_0, for laminar flow of the fluid through uniform granular media, can be derived from the Kozeny-Carman equation, if the flow rate, water temperature, media size and

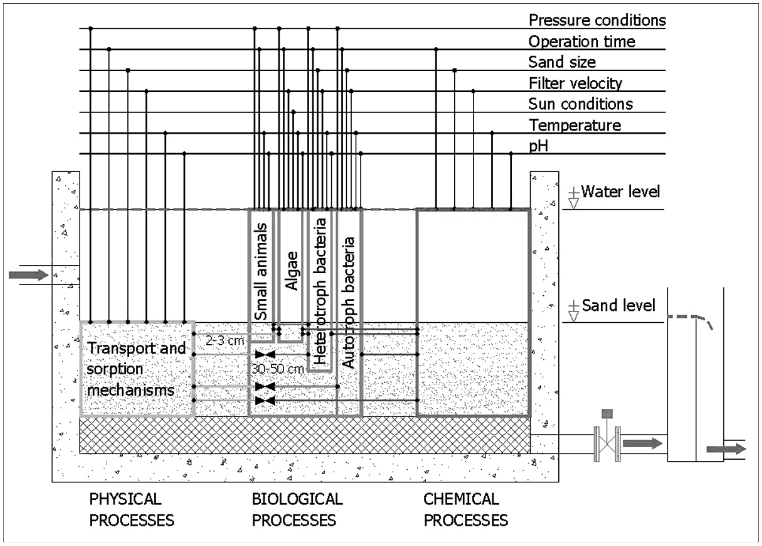


Figure 1. *System model for slow sand filters.*

porosity are known (Camp 1964, Ives 1969, Deb 1969, Öllös 1987).

$$h_0 = j \cdot \psi_0^2 \cdot \nu/g \cdot (1 - \varphi_0)^2 / \varphi_0^3 \cdot v/d_0^2 \quad (15\,a)$$

where:

- j is a dimensionless constant
- ψ_0 is a dimensionless shape factor of grains in clean bed
- φ_0 is the porosity of clean bed (%).
- d_0 is the grain diameter in clean bed (m)

The total pressure losses can then be calculated as:

$$H_0 = {}_0\int^L h_0 \, dL \quad (15\,b)$$

where:

- L is the total thickness of the filter media (m)

After the start of the filter run, the head-loss within a slow sand filter is caused by flow through the upper thin layer, Schmutzdecke, and the sand bed. As the filter is operated the schmutzdecke develops and its hydraulic resistance increase, causing most of the head-losses.

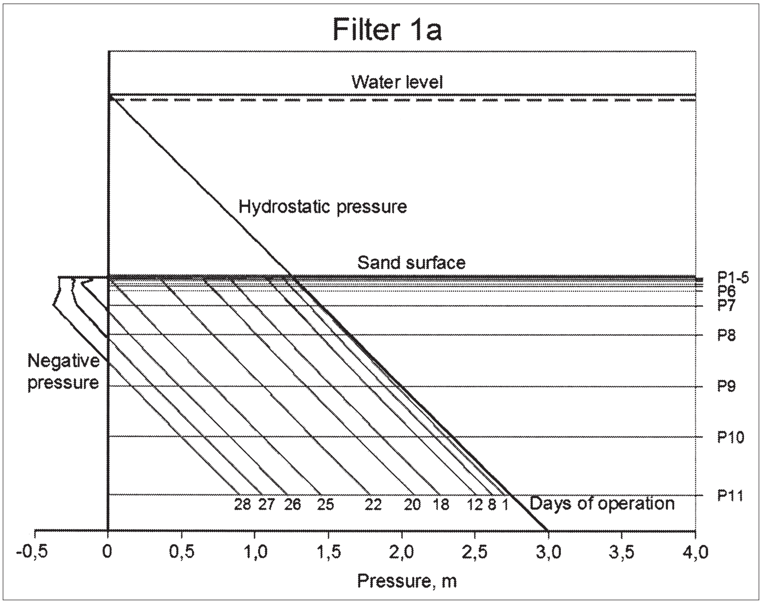


Figure 2. *Pressure conditions in filter 1a during test 1.*

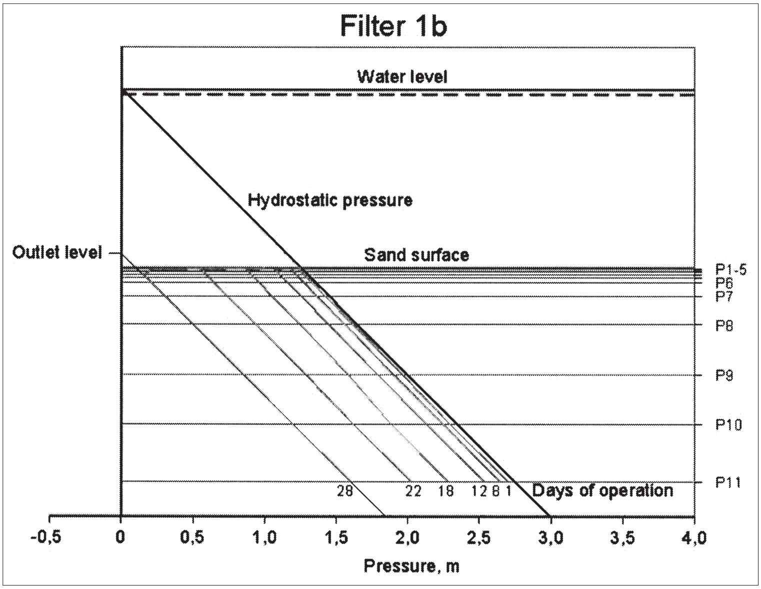


Figure 3. *Pressure conditions in filter 1b during test 1.*

The Schmutzdecke is defined as a thin slimy layer of both deposited and synthesized material, largely organic in origin, on the top of the filter bed (Huisman and Wood 1974, Barret et al 1991, Öllös 1998).

Fig 2 and fig 3 show the head-losses in a pilot slow sand filter measured by 11 piezometers at different depths. The tests show that the pressure losses are concentrated to the top of the sand, mainly to the schmutzdecke. The depth of this active layer is about 1–5 cm, depending on filtration velocity, sand characteristics, raw water quality and weather conditions. Under this level the sand remains almost hydraulically clean, i.e. impurities exist, but these will affect the pressure losses marginally after a few years of operation. The clean-bed losses are in general less then 10 cm (Jabur and Mårtensson 1999, Jabur and Mårtensson 2003).

The occurrence of negative pressure at unsuitable operation conditions of slow sand filters is also demonstrated in fig 2. The negative pressure occurs when the pressure level in the sand is below the atmospheric pressure. The development of negative pressure with increasing depth in the sand went relatively quick, fig 4 (Jabur and Mårtensson 1999).

The negative pressure will affect both the hydraulic

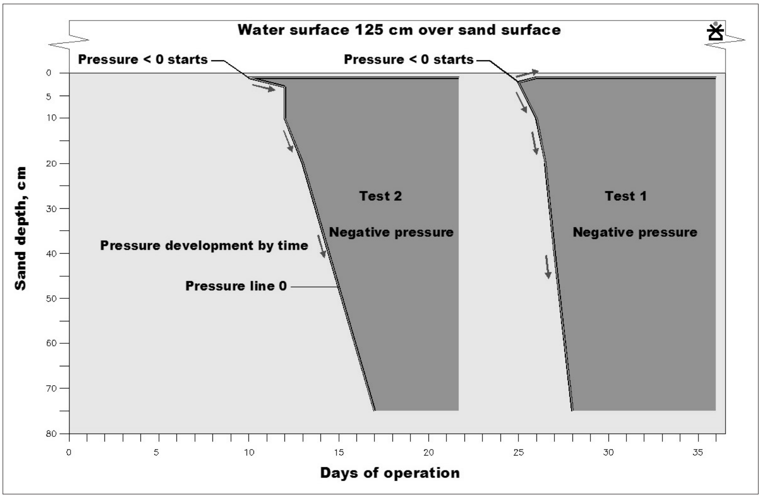


Figure 4. *Development of negative pressure in filter 1a in test 1 and 2.*

Figure 5. *Craters caused by gas release at negative pressure.*

capacity of the filter and the filtered water quality (Huisman and Wood 1974, Barret et al 1991). It causes a formation of gas bubbles and air binding in the pores of the media, which result in rapid increase in filter resistance, fig 5–6 (Jabur and Mårtensson 2003). The effect is usually an initial rapid reduction and then large fluctuations in the filter velocity.

Negative pressure generally ruins the water quality, which was shown in form of e.g. coliform bacteria, algae and other micro organisms (Jabur and Mårtensson 1999, Jabur et al 2002).

The negative pressure can be avoided by regulation of the outlet of the filters. Generally the outlet level should be above the sand surface to completely eliminate the risk of forming local vacuum (Jabur and Mårtensson 1999).

Developments

Today equipment is available on the market that relatively easy counts the number of particles in water. Given the parameter C an opportunity opens to determine some of the key parameters in the theory, for a certain sand and raw water quality.

Sand to be used in slow sand filters is usually following strict specifications. A given sand quality has certain porosity, a certain sand curve and shape and even the included minerals are given. For a given raw water, after pre-treatment, it is thus easier to empirically determine several filtration parameters as λ_0, λ, φ_0, φ, σ, d_0, d. Out of this it is possible to estimate, perhaps guided by the organic content in the suspended solids, σ_2. The remaining unknown parameters a and S in equations 13–14 can then be determined.

A number of tests can gradually result in relations that describe the variation of S by time and depth for the conditions at a certain water works. By comparing the results from many water works it might be possible to find limits and typical values for the filter parameters that can be used in modelling, as design parameters when predicting operation time, deep cleaning interval etc.

Conclusions

Reliable mathematical models that can describe the kinetic behaviour of slow sand filters are at present absent. A lot of research remains before physical, chemical and especially biological processes can be described mathematically, either by developing the existing filtration models, or by other approaches.

Empirical values of some parameters might be derived from repeated tests. These results can be used to gradually refine mathematical models.

The importance of these attempts to mathematically model the slow sand filters for a quantitative description of the process is obvious. This could provide an aid for better understanding of these processes and for rational design and operation criteria.

List of symbols

A Cross-section (m^2)

a Filtration parameter (cm^{-1}). Represent the positive effect of deposits on the filter efficiency and water quality early in the filtration process.

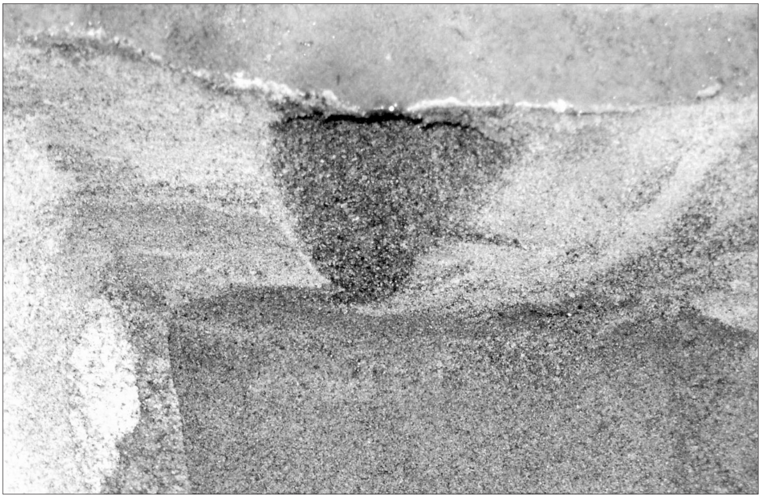


Figure 6. *Very coarse material concentrated in the crater, below moist dark sand. Fine material found in a ring around the crater.*

b Filtration parameter (cm^{-1}). Represent the negative effect of deposits on the filter efficiency and water quality late in the filtration process.
c Constant in Iwasaki equation, similar to filtration parameter a (cm^{-1})
C Concentration of particles (in number of particles/cm^3)
d Grain diameter (m)
d_0 Grain diameter of clean bed (m)
g The acceleration of gravity (m/s^2)
h_w Thickness of water layer, piezometric head (m)
h_0 Pressure losses in clean bed (m/m)
I Hydraulic gradient (the slope of the piezometric level)
I_C Critical hydraulic gradient
I_0 Limitation gradient, highest gradient at which v = 0
j Kozeny-Carman dimensionless constant
k Coefficient of permeability (m/s)
L Distance from top of filter bed from which e.g. C is measured (m)
φ Porosity (%)
φ_0 Porosity of clean bed (%)
p Pressure in general
Q Flow (m^3/s)
Re Reynolds number
S Filter parameter (cm^{-1}). Represent the specific SSF processes in fig. 1.
t Time in general, filtration time
v Flow velocity (m/s) (approach velocity), $= Q/A = p \cdot v'$
v' Real velocity (m/s)
z The height of the fluid above the object (m)
α Surface area per grain divided by the square of the grain size d (m^2/m^2).
β Volume per grain divided by the cube of the grain size d (m^3/m^3).
ν Kinematical water viscosity (m^2/s).
σ Specific dimensionless deposit parameter, vol. deposit/bed volume, $\sigma = f(t,L)$
σ_1 Inconvertible dimensionless specific deposit parameter
σ_2 Convertible dimensionless specific deposit parameter
λ Filter coefficient or impediment modulus (cm^{-1}), $\lambda = f(t,L)$
λ_0 Initial filter coefficient (cm^{-1})
ψ Shape factor of grains = α/β
ψ_0 Shape factor of grains in clean bed = α/β

References

Arun K. Deb (1969): Modern Theory of Sand filtration. Jour. of Sanitary Engineering Division. ASCE. Vol. 95, No. SA3, June.

Avner A. and Menahem R., (1977): A Model to Predict Concentration and Head-loss Profiles in Filtration. J. AWWA. Vol. 69, No. 8.

Barret J. M., Collins M. R., Janonis B. A., Logdson G. S. (1991): Manual for Design of Slow Sand Filtration. Edited by Hendricks D. AWWA Research Foundation, Denver.

Huisman L. and Olsthoorn T. N. (1983). Artificial Groundwater Recharge. Pitman Publishing INC. Massachusetts.

Huisman L. and Wood W. E. (1974): Slow Sand Filtration. WHO. Geneva.

Ives K. J. (1969). Theory of Filtration. Special Subject No 7. International Water-Supply Congress and Exhibition. Vienna.

Jabur H. (1976): Slow Sand Filters and its Role in Water Treatment. PhD thesis. Hungarian Academy of Sciences.

Jabur H. and Mårtensson J. (1999): Optimering av långsamfilter. VA-Forsk No 17-99.

Jabur H. And Mårtensson J. (2003): Rensningsmetoder för Långsamfilter. VA-Forsk No 39-03.

Jabur H., Mårtensson J., Persson K. (2002): Synpunkter på Mikroorganismer som idikatorer i vattenförsörjning. Vatten 58, Lund.

Kovacs Gy. (1972): A Szivargas Hidraulikaja. Akademiai Kiado. Budapest.

Matsui Y. and Tambo N. (1995): Mathematical Description of Deep-Filter Performance. Aqua. Vol. 44, No 5.
Mintz D. M. (1966): Modern Theory of Filtration. Special Subject No 10. International Water-Supply Congress and Exhibition. Barcelona.
Ojha C. S. P . and Graham N. J. D. (1996): Numerical Assessment of Microbial Interaction in Slow Sand Filtration Modelling. Advances in Slow Sand and Alternative Biological Filtration. Edited by N. J. D. Graham and Collins. Jon Willy & Sons.
Öllös G. (1987): Vizellatas. Aqua Kiado. Budapest.
Öllös G. (1998): Vizellatas- Uzemeltetes. OVF. Egri Nyomda Kft. Budapest.
Thomas R. Camp (1964): Theory of Water Filtration. Jour. of Sanitary Engineering Division. ASCE. Vol. 90, No. SA4, August.
Tomihisia Iwasaki (1937): Some Notes on Sand Filtration. AWWA. Vol. 39, No. 10.
Woodward C. A. and Ta C. T. (1988): Development in modelling in Slow Sand Filtration. Slow Sand Filtration: Recent Development in Water Treatment Technology. Edited by Graham N. J. D. Chichester, U. K. Ellis Horwood Ltd.